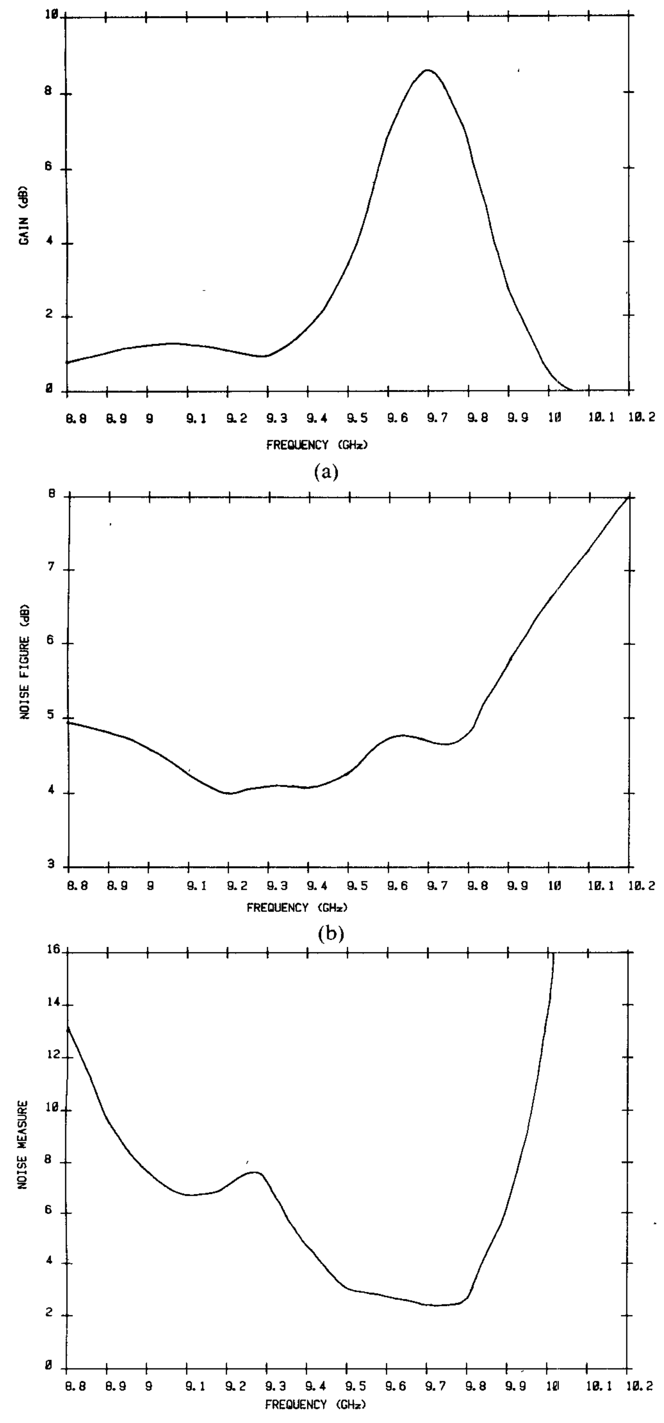

Fig. 5. Measured performance of the experimental amplifier. (a) Gain versus frequency. (b) Noise figure versus frequency. (c) Noise measure versus frequency.

This fact, combined with the difficulty of accurately measuring the phase of reflection coefficients, suggests that the discrepancy between predicted and measured noise measure is reasonable.

IV. Conclusion

The loci of constant noise measure presented here provide a useful graphical representation of how this quantity varies with source reflection coefficient (e.g., Fig. 3). The equations describing these circles have also been used to determine the minimum noise measure and associated optimum source termination for a given active device. The advantages of the new equations, when compared with existing methods of minimizing the noise measure, lie in the fact that they utilize reflection coefficient parameters which are readily available for most modern transistors. Furthermore, they yield results which are consistent with Smith Chart design techniques, thereby allowing graphical comparisons to be made with other criteria such as gain and noise figure.

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Graph Transformations of Nonuniform Coupled Transmission Line Networks and Their Application

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***Abstract*—The graph transformation method of [5] has been extended to apply for a class of coupled nonuniform transmission lines whose self and mutual line constants have the same functional dependence along the**

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direction of propagation, and it is assumed that the mode of propagation is TEM. First, the graph representation of such nonuniform coupled two-wire transmission lines is derived by decomposition of the 4×4 admittance matrix. This leads to three-port equivalent circuits of nonuniform coupled two-wire line networks. Then, the multiport graph transformations of networks consisting of nonuniform transmission lines and nonuniform stubs are shown. By using the graph transformation of *n*-wire nonuniform coupled lines, the equivalent circuits for the nonuniform interdigital line and the nonuniform meander line are given. Finally, a meander-line low-pass filter consisting of parabolic tapered coupled transmission lines designed on this equivalent circuit is shown.

I. Introduction

Coupled transmission-line networks are extremely important in microwave network theory and are used extensively in all types of microwave components: filters, couplers, matching sections, and equalizers [1]–[9]. The graph transformation method [5] is a very powerful technique for the analysis and synthesis of coupled transmission-line networks. By using this technique, we can analyze coupled transmission lines without troublesome matrix operations. Nonuniform coupled transmission lines may show superior transmission responses when compared with uniform ones. We have analyzed a class of two-wire nonuniform transmission lines and have reported some useful equivalent representations of nonuniform transmission lines [12]–[14].

In this paper, we deal with general nonuniform coupled transmission lines whose self and mutual characteristic immitance distributions vary at the same rate, and we discuss the graph transformation method for these circuits. First, it is shown that nonuniform coupled two-wire transmission lines may be represented by the circuit consisting of uncoupled nonuniform transmission lines and nonuniform short-circuited stubs with the same taper. Second, three-port equivalent circuits of nonuniform coupled two-wire line networks are shown. Then, multiport graph transformations of networks consisting of nonuniform transmission lines and nonuniform stubs are given. By using the graph transformation of n-wire nonuniform coupled lines, equivalent circuits for the nonuniform interdigital and nonuniform meander line are presented. Finally, a meander-line low-pass filter consisting of parabolic tapered coupled transmission lines is designed by using this equivalent circuit.

II. Graph Representations of Nonuniform Coupled Two-Wire Line Networks

We define the characteristic admittance distribution $Y(x)$ of a general lossless nonuniform transmission line as follows:

$$Y(x) = y_0 \cdot f(x) \qquad (0 \leqq x \leqq l) \tag{1}$$

where x is the distance along the line, l is the line length of the line, y_0 is the characteristic admittance of nonuniform transmission line at $x = 0$, and $f(x)$ is the taper coefficient. This nonuniform transmission line may be described by the chain matrix equation

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} A(x) & \frac{1}{y_0}B(x) \\ y_0 C(x) & D(x) \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} \tag{2}$$

where $A(x)$, $B(x)$, $C(x)$, and $D(x)$ are elements of the chain matrix, and satisfy the following condition:

$$A(x)\cdot D(x) - B(x)\cdot C(x) = 1. \tag{3}$$

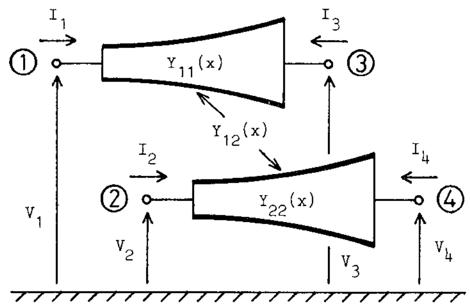


Fig. 1. General lossless nonuniform coupled two-wire transmission lines

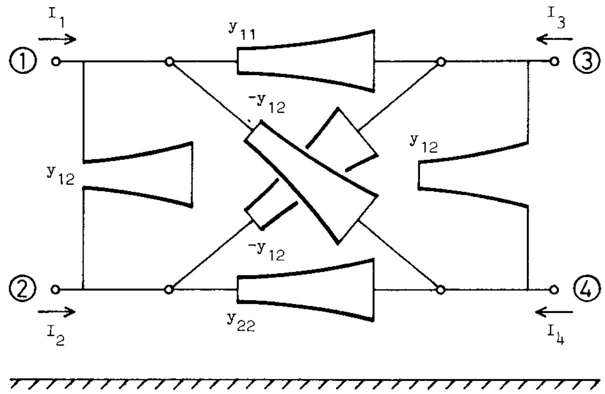


Fig. 2. Equivalent circuit of nonuniform coupled two-wire transmission lines shown in Fig. 1.

Driving point admittances of nonuniform open- and short-circuited stubs as defined by Y_f and Y_s, respectively, are given by

$$Y_f = y_0 \frac{C(x)}{A(x)} \quad Y_s = y_0 \frac{D(x)}{B(x)} \tag{4}$$

$$Y_f^* = y_0 \frac{C(x)}{D(x)} \quad Y_s^* = y_0 \frac{A(x)}{B(x)}. \tag{5}$$

Because of the variation of characteristic immitance distributions, there are four types of single nonuniform transmission-line stubs [14]. Here, we call the stubs of Y_f and Y_s the normal-type and the stubs of Y_f^* and y_s^* are the reverse-type.

Then, we define the self and mutual characteristic admittance distributions of general lossless nonuniform coupled two-wire transmission lines, shown in Fig. 1, as follows:

$$[Y(x)] = \begin{bmatrix} Y_{11}(x) & -Y_{12}(x) \\ -Y_{12}(x) & Y_{22}(x) \end{bmatrix} = \begin{bmatrix} y_{11}f(x) & -y_{12}f(x) \\ -y_{12}f(x) & y_{22}f(x) \end{bmatrix} = [Y(0)]\cdot f(x) \tag{6}$$

$$[Y(0)] = \begin{bmatrix} y_{11} & -y_{12} \\ -y_{12} & y_{22} \end{bmatrix}. \tag{7}$$

where

$Y_{ii}(x)$ self characteristic admittance distribution of ith transmission line ($i = 1,2$),

$Y_{12}(x)$ mutual characteristic admittance distribution between the first and second transmission lines,

TABLE I
THREE-PORT EQUIVALENT CIRCUITS OF NONUNIFORM COUPLED TWO-WIRE LINE NETWORKS

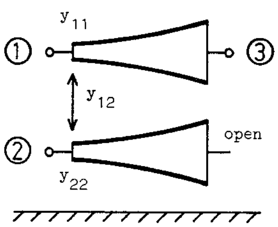
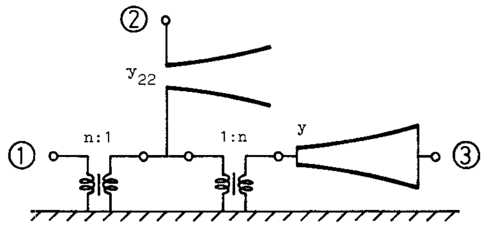
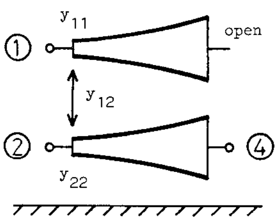
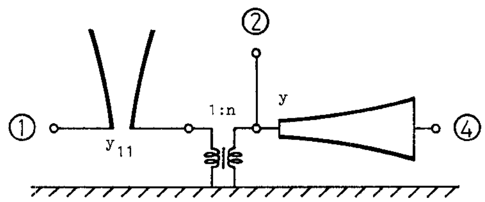
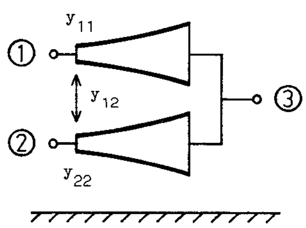
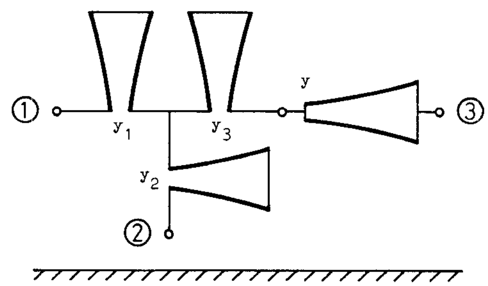
	ORIGINAL CIRCUIT	EQUIVALENT CIRCUIT	FORMULA
1			$n = \frac{y_{22}}{y_{12}}$ $y = y_{11} - \frac{y_{12}^2}{y_{22}}$
2			$n = \frac{y_{11}}{y_{12}}$ $y = y_{22} - \frac{y_{12}^2}{y_{11}}$
3			$y_1 = \Delta y/(y_{22} - y_{12})$ $y_2 = \Delta y/(y_{11} - y_{12})$ $y_3 = -(y_1 + y_2)$ $y = y_{11} + y_{22} - 2y_{12}$ $\Delta y = y_{11}y_{22} - y_{12}^2$

TABLE II
TWO-PORT NONUNIFORM DISTRIBUTED NETWORK TRANSFORMATIONS

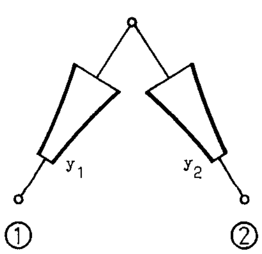
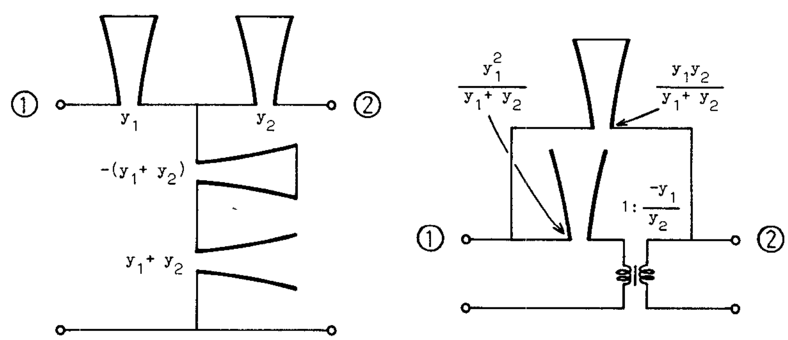
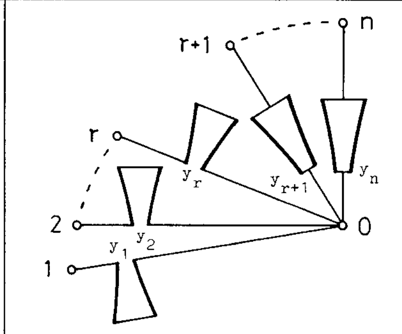
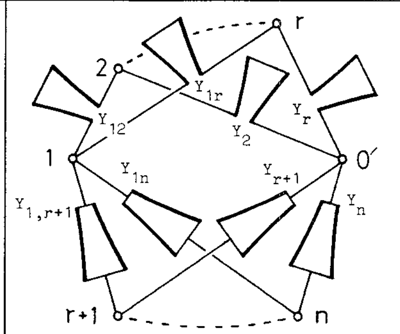
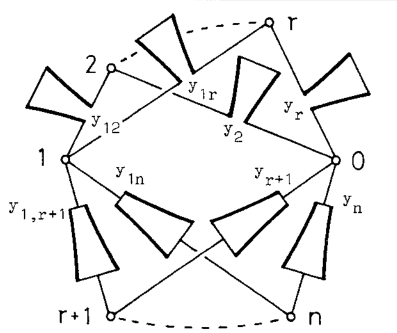
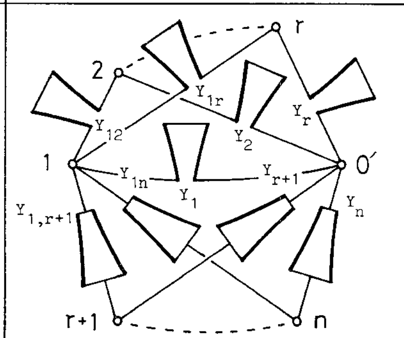
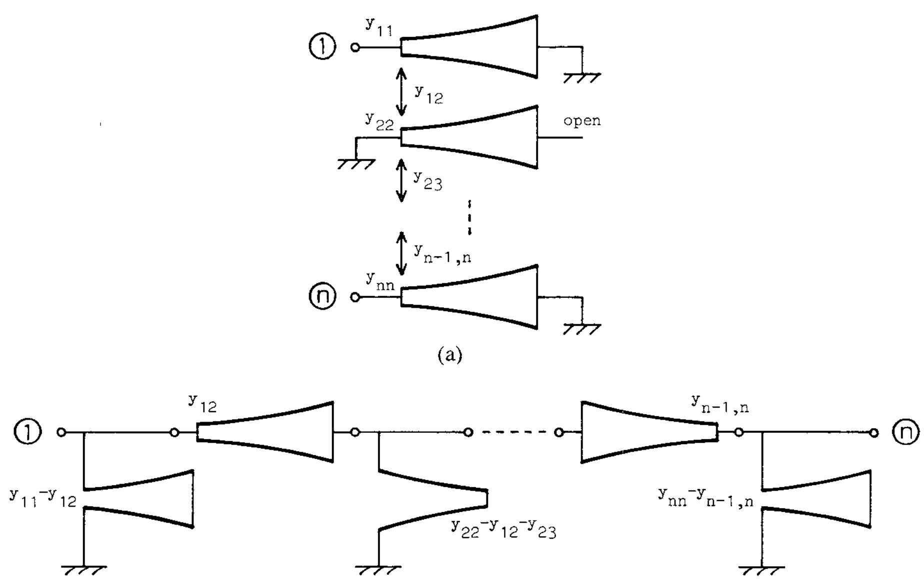
ORIGINAL CIRCUIT	EQUIVALENT CIRCUIT
	

TABLE III
MULTIPORT NONUNIFORM DISTRIBUTED NETWORK TRANSFORMATIONS

	ORIGINAL CIRCUIT	EQUIVALENT CIRCUIT	FORMULA
1			$Y_s = y_s \frac{y_0'}{y_0}$ (s=2,3,---,n) $Y_{1s} = \frac{y_1 y_s}{y_0}$ $y_0 = \sum_{s=1}^{n} y_s$ $y_0' = y_0 - y_1$
2			Y_{1n} = Arbitrary Number $Y_1 = \frac{(y_{1n}-Y_{1n})(y_{1n}-Y_{1n}+y_n)}{y_n^2} y_0'$ $y_0' = \sum_{s=2}^{n} y_s$ $Y_s = y_s \frac{(y_{1n}-Y_{1n}+y_n)}{y_n}$ $Y_{1s} = y_{1s} - y_s \frac{y_{1n}-Y_{1n}}{y_n}$ (s=2,3,---,n)



(a)

(b)

Fig. 3. Nonuniform interdigital line in which couplings beyond nearest neighbors are negligible and its equivalent circuit.

y_{ii} self characteristic admittance of ith transmission line at $x = 0$ ($i = 1,2$), and

y_{12} mutual characteristic admittance between the first and second transmission lines at $x = 0$.

Namely, we deal with general nonuniform coupled transmission lines whose self and mutual characteristic admittance distributions vary at the same taper coefficient $f(x)$ defined in (1).

These coupled transmission lines may be described by the chain matrix equation

$$\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} [I]A(x) & [Y(0)]^{-1}B(x) \\ [Y(0)]C(x) & [I]D(x) \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ -I_3 \\ -I_4 \end{bmatrix} \quad (8)$$

where $[I]$ is the 2×2 identity matrix. Consequently, the following admittance matrix equation is obtained:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} [Y(0)]\dfrac{D(x)}{B(x)} & -[Y(0)]\dfrac{1}{B(x)} \\ -[Y(0)]\dfrac{1}{B(x)} & [Y(0)]\dfrac{A(x)}{B(x)} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \equiv [Y]\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}. \tag{9}$$

By decomposing the admittance matrix of (9) [5], the nonuniform coupled two-wire transmission lines shown in Fig. 1 may be represented as the circuit consisting of uncoupled nonuniform transmission lines and nonuniform short-circuited stubs shown in Fig. 2.

III. Three-Port Equivalent Circuits

Three-port equivalent circuits of nonuniform coupled two-wire line networks are also a very powerful technique for the analysis of n-wire line networks. These are introduced in Table I. The networks which have one open-circuited terminal and their three-port equivalent circuits are shown in Table I. The network of which terminals 3 and 4 are connected by a zero-length line and its equivalent circuit are alos shown in Table I.

IV. Graph Transformations of Nonuniform Transmission-Line Networks

By using a similar technique shown in [5], we can obtain the equivalent circuit of two-stage cascaded nonuniform transmission lines and the transformation of a multiport nonuniform distributed network. Table II shows the equivalency of the two-stage cascaded nonuniform transmission lines and Table III shows the transformation of a multiport nonuniform distributed circuit.

V. Application of Graph Transformations

A. Nonuniform Interdigital Line and Nonuniform Meander Line

Equivalent circuits of Fig. 2 and Tables II and III are very useful for the derivation of equivalent circuits of nonuniform coupled transmission lines. Here, we show two examples.

Example 1 is the nonuniform interdigital line of Fig. 3(a) in which couplings beyond the nearest neighbors are negligible. In this case, by using the graph transformation of n-wire nonuniform coupled lines, we can obtain the final equivalent circuit of Fig. 3(b).

Example 2 is the nonuniform meander line of Fig. 4(a) in which couplings between the adjacent conductors only are considered. The equivalent circuit of Fig. 4(b) is obtained in the same manner as for the case of Example 1.

B. Nonuniform Meander-Line Low-Pass Filter

Using these equivalent circuits, we may easily discuss the design of transmission circuits consisting of nonuniform coupled transmission lines. Here, we show one design method of the nonuniform meander-line low-pass filter.

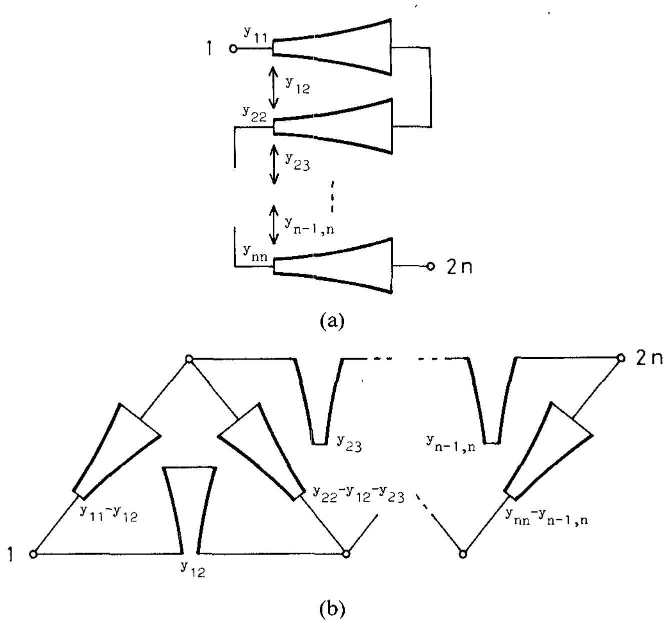


Fig. 4. Nonuniform meander line in which couplings beyond nearest neighbors are negligible and its equivalent circuit.

TABLE IV
Element Values of Uniform Meander Line

	Max Flat		Chebyshev	
b_0	2	3	2	3
y_{11}	0.7689	0.7915	0.3788	0.3838
y_{22}	1.9350	2.1090	1.1890	1.2820
y_{12}	0.1796	0.2842	0.0929	0.1431

$$\text{Specification}\begin{cases}\text{Cutoff frequency: } f_c = f_0/2 \\ \text{Maximum attenuation in the passband: 1 dB}\end{cases}$$

where f_0 is the frequency at which the line length l of a uniform line is a $1/4$ wavelength, and the internal resistances of the source and the load are unity.

The design method of the uniform meander line is shown in [9]. For example, for the case of element number $n = 3$, the operating transfer constant $S(p)$ with maximally flat response and Chebyshev response, respectively, are given by

$$|S(p)|^2_{\mathrm{MF}} = 1 + \epsilon^2 \frac{-p^6}{(1-p^2)(b_0 - p^2)^2} \tag{10}$$

$$\left.\begin{aligned} &|S(p)|^2_{\mathrm{Cheby}} = 1 + \epsilon^2 \frac{-k_0^2 p^2 (k_1 + p^2)^2}{(1-p^2)(b_0-p^2)^2} \\ &b_0 = 2,\ k_0 = 11.970,\ k_1 = 0.64556 \\ &b_0 = 3,\ k_0 = 16.828,\ k_1 = 0.66384 \end{aligned}\right\} \tag{11}$$

where p is Richard's variable, b_0 is the coupled constant, and ϵ is a constant. Element values of the uniform meander line are

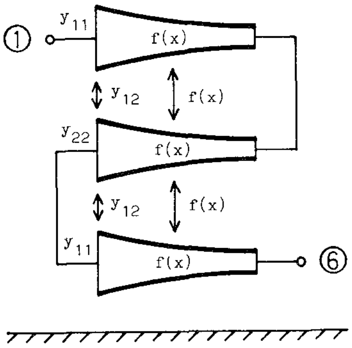


Fig. 5. Nonuniform meander line consisting of parabolic tapered coupled lines.

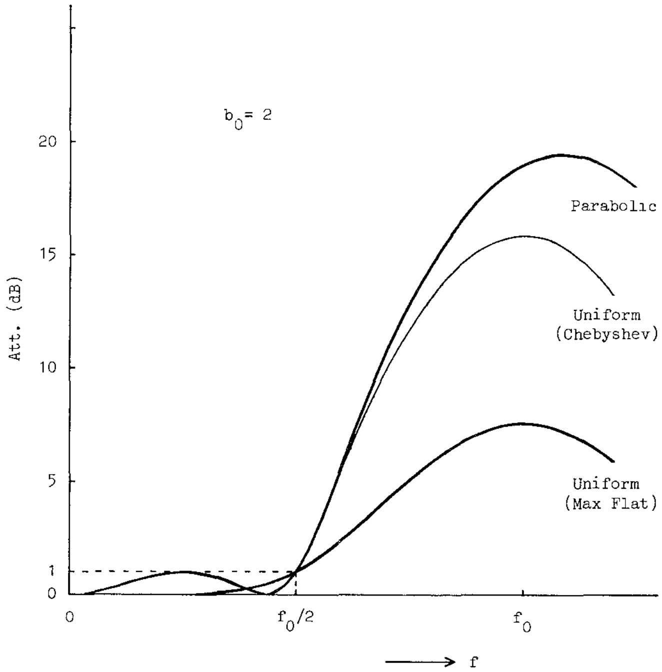


Fig. 6. Attenuation characteristics of meander-line low-pass filters

obtained as shown in Table IV [9]. For simplicity, we choose parabolic tapered coupled transmission lines whose taper coefficient $f(x)$ is given by

$$f(x)=\left(1-a\frac{x}{l'}\right)^2 \qquad (0<a<1, 0\leqq x\leqq l') \tag{12}$$

where l' is the line length of the parabolic tapered line.

We set the values of the maximally flat response in Table IV to element values at $x=0$ of the parabolic tapered meander line shown in Fig. 5. Then, by calculating the taper coefficient $f(x)$ and the line length l' to satisfy the specification, these values are obtained as follows.

In the case of $b_0=2$

$$f(x)=\left(1-0.483\frac{x}{l'}\right)^2, \qquad l'=0.956l. \tag{13}$$

In the case of $b_0=3$

$$f(x)=\left(1-0.462\frac{x}{l'}\right)^2, \qquad l'=0.936l \tag{14}$$

where l is the line length of the uniform line.

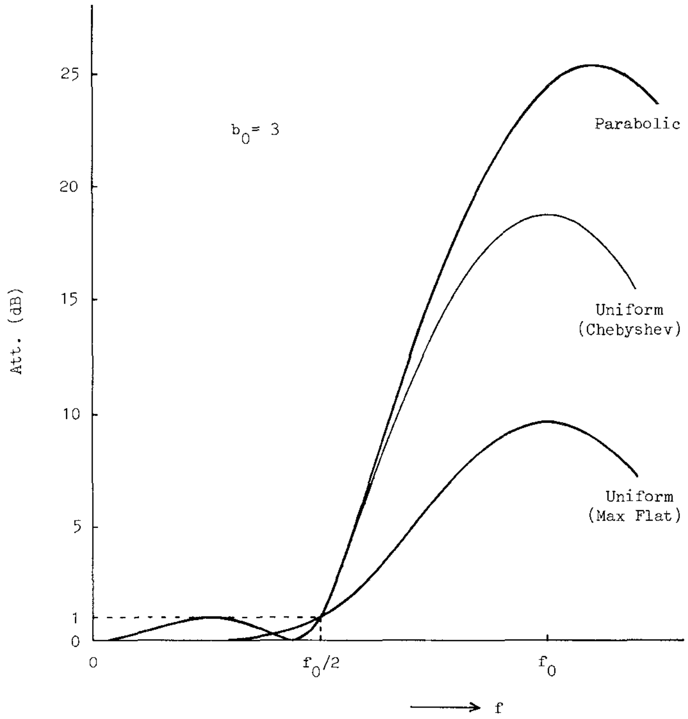


Fig. 7. Attenuation characteristics of meander-line low-pass filters.

Figs. 6 and 7 show the attenuation characteristics of the uniform meander line and the parabolic tapered meander line. As is seen from these figures, the parabolic tapered meander line gives larger attenuation in the stopband as compared to the uniform meander line. The line length of the parabolic tapered line is shorter than the one of uniform line.

VI. Conclusion

We have obtained graph transformations of general nonuniform coupled transmission lines whose self and mutual characteristic immitance distributions vary at the same rate. These nonuniform coupled two-wire transmission lines can be expressed as the circuit consisting of uncoupled nonuniform transmission lines and nonuniform short-circuited stubs with the same taper. Then, three-port equivalent circuits of nonuniform coupled two-wire line networks, and multiport graph transformations for nonuniform distributed circuits are given. By using graph representations and three-port equivalent circuits, two-port equivalent circuits for nonuniform coupled transmission lines, including the nonuniform interdigital and meander lines, may be easily obtained. Finally, one design method for meander-line low-pass filter consisting of parabolic tapered coupled transmission lines is shown. Transmission responses of nonuniform meander line are superior when compared with uniform ones.

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